1. **Найти разложения Шеннона по указанным подмножествам переменных следующих формул:**

**Показать равносильность полученных формул тождественными преобразованиями формул или таблицами истинности.**

**x ⊕ yz → (x ↔ z) по {y} и {x, y, z}**

**Разложение по {y}:**

**F(x, y, z) = ~y(x, 0, z) V y(x, 1, z) = ~y(x ⊕ 1) V y(x ⊕ z -> (x <-> )) =**

**= ~y(x ⊕ 1) V y(x ⊕ ~z V (~x~z V xz)) = ~x~y V y(x ⊕ ~z V (~x~z V xz)) =**

**= ~x~y V y(x ⊕ (~z V xz)) = ~x~y V y(~x(~z V zx) V x ~(z V xz)) =**

**= ~x~y V y(~x~z) V xz ~(xz) = ~x ~y V ~xy~z V xz(~x V ~z) =**

**= ~x~y V ~xy~z**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **x** | **y** | **z** | **x ⊕ yz → (x ↔ z)** | **~x~y V ~xy~z** |
| **0** | **0** | **0** | **1** | **1** |
| **0** | **0** | **1** | **1** | **1** |
| **0** | **1** | **0** | **1** | **1** |
| **0** | **1** | **1** | **0** | **0** |
| **1** | **0** | **0** | **0** | **0** |
| **1** | **0** | **1** | **0** | **0** |
| **1** | **1** | **0** | **0** | **0** |
| **1** | **1** | **1** | **0** | **0** |

**Разложение по {x, y, z}:**

**F(x, y, z) = ~x~y~z(0, 0, 0) V ~x~yz(0, 0, 1) V ~xy~z(0, 1, 0) V ~xyz(0, 1, 1) V**

**V x~y~z(1, 0, 0) V x~yz(1, 0, 1) V xy~z(1, 1, 0) V xyz(1, 1, 1) =**

**= ~x~y~z V ~x~yz V ~xy~z**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **x** | **y** | **z** | **x ⊕ yz → (x ↔ z)** | **~x~y~z V ~x~yz V ~xy~z** |
| **0** | **0** | **0** | **1** | **1** |
| **0** | **0** | **1** | **1** | **1** |
| **0** | **1** | **0** | **1** | **1** |
| **0** | **1** | **1** | **0** | **0** |
| **1** | **0** | **0** | **0** | **0** |
| **1** | **0** | **1** | **0** | **0** |
| **1** | **1** | **0** | **0** | **0** |
| **1** | **1** | **1** | **0** | **0** |

**(x → z) ⊕ (x ∨ y) по {z} и {y, z}**

**Разложение по {z}:**

**F(x, y, z) = z((x → 1) ⊕ (x ∨ y)) v ~z((x → 0) ⊕ (x ∨ y)) =**

**= z(1 ⊕ (x ∨ y)) v ~z(~x ⊕ (x ∨ y)) =**

**= z(1 ⊕ (x ∨ y)) v ~z(~x ⊕ (x ∨ y)) = z(~(x ∨ y)) v ~z(~x(~(x v y)) v x(x v y))**

**= z(~x~y ) v ~z(~x~y v x v xy)) = z~x~y v ~z~x~y v ~zx v ~zxy**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **x** | **y** | **z** | **(x → z) ⊕ (x ∨ y)** | **z~x~y v ~z~x~y v ~zx v ~zxy** |
| **0** | **0** | **0** | **1** | **1** |
| **0** | **0** | **1** | **1** | **1** |
| **0** | **1** | **0** | **0** | **0** |
| **0** | **1** | **1** | **0** | **0** |
| **1** | **0** | **0** | **1** | **1** |
| **1** | **0** | **1** | **0** | **0** |
| **1** | **1** | **0** | **1** | **1** |
| **1** | **1** | **1** | **0** | **0** |

**Разложение по {y, z}:**

**F(x, y, z) =**

**yz((x → 1) ⊕ (x ∨ 1)) v y~z((x → 0) ⊕ (x ∨ 1)) v ~yz((x → 1) ⊕ (x ∨ 0)) v ~y~z((x → 0) ⊕**

**⊕ (x ∨ 0)) = yz(1 ⊕ 1 ) v y~z(~x ⊕ 1 ) v ~yz(1 ⊕ x ) v ~y~z(~x ⊕ x) =**

**= y~zx v ~yz~x v ~y~z**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **x** | **y** | **z** | **(x → z) ⊕ (x ∨ y)** | **y~zx v ~yz~x v ~y~z** |
| **0** | **0** | **0** | **1** | **1** |
| **0** | **0** | **1** | **1** | **1** |
| **0** | **1** | **0** | **0** | **0** |
| **0** | **1** | **1** | **0** | **0** |
| **1** | **0** | **0** | **1** | **1** |
| **1** | **0** | **1** | **0** | **0** |
| **1** | **1** | **0** | **1** | **1** |
| **1** | **1** | **1** | **0** | **0** |

1. **Приведением формулы к базису { , \* , ∨ } построить ДНФ, аналитически определить СДНФ, найти сокращенную ДНФ алгоритмом Квайна-МакКласки.**

**(xy ⊕ z) → x ∨  = (~x~yz V xy~z) -> x~z V ~y = (z~x V z~y V xy~z) -> x~z V ~y =**

**x ∨ ~y ∨ ~z**

**СДНФ**

**x(yV~y)(zV~z) V ~y(xV~x)(zV~z) V ~z(xV~x)(yV~y) = xyz V xy~z V x~yz V x~y~z V x~yz V V x~y~z V~x~yz V ~x~y~z V xy~z V x~y~z V ~xy~z V ~x~y~z = xyz V xy~z V x~yz V x~y~z V ~x~yz V ~x~y~z V ~xy~z**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **#** | **X** | **Y** | **Z** |  |  |
| **G1** | **1** | **0** | **0** | **0** | **\*** |  |
| **G2** | **2** | **0** | **0** | **1** | **\*** |  |
| **3** | **0** | **1** | **0** | **\*** |  |
| **4** | **1** | **0** | **0** | **\*** |  |
| **G3** | **5** | **1** | **0** | **1** | **\*** |  |
| **6** | **1** | **1** | **0** | **\*** |  |
| **G4** | **7** | **1** | **1** | **1** |  |  |
| **G5** | **8** | **0** | **0** | **x** | **\*** | **(1,2)** |
| **9** | **0** | **X** | **0** | **\*** | **(1,3)** |
| **10** | **X** | **0** | **0** | **\*** | **(1,4)** |
| **G6** | **11** | **X** | **0** | **1** | **\*** | **(2,5)** |
| **12** | **X** | **1** | **0** | **\*** | **(3,6)** |
| **13** | **1** | **0** | **x** | **\*** | **(4,5)** |
| **14** | **1** | **x** | **0** | **\*** | **(4,6)** |
| **G7** | **15** | **1** | **X** | **1** | **\*** | **(5,7)** |
| **16** | **1** | **1** | **x** | **(** | **(6,7)** |
|  | **17** | **x** | **0** | **x** |  | **(8, 13)** |
|  | **18** | **X** | **X** | **0** |  | **(9, 14)** |
|  | **19** | **X** | **0** | **x** |  | **(10,11)** |
|  | **20** | **1** | **x** | **x** |  | **(14,15)** |

**F = x V ~y V ~z**

1. **Приведением формулы к базису { , \* , ∨ } построить ДНФ, аналитически определить СДНФ, найти сокращенную ДНФ алгоритмом Блейка-Порецкого.**

**Для полученных СДНФ и сокращенной ДНФ построить таблицу Квайна и определить кратчайшие и минимальные ДНФ булевой функции.**

**Получить полином Жегалкина.**

**ДНФ:**

**(x | ) ∨ (y | )x = ~(x ~y) V ~(y~z)x ~z = ~x V y V (~y V z)x ~z =**

**= ~x V y V x~y~z V 0 = ~x V y V x~y~z**

**СДНФ:**

**~x(y v ~y)(~z v z) V y(~x V x)(~z V z) V x~y~z = ~xy~z V ~x~y~z ~xyz V ~x~yz V xy~z V xyz V x~y~z**